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SCREENING LENGTHS IN SU(2) GAUGE THEORY AT FINITE TEMPERATURE^a

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We discuss the calculation of electric and magnetic screening masses in SU(2) gauge theory. The temperature dependence of these masses obtained from the long-distance behaviour of spatial correlation functions has been analyzed for temperatures up to $10^4 T_c$.

1 Introduction

One of the basic concepts that guide our intuition about the properties of the high temperature plasma phase of QCD is the occurrence of chromo-electric and -magnetic screening. From the early perturbative calculations at high temperatures¹ we know that a non-vanishing electric screening mass, m_e , is needed to control the infrared behaviour of QCD at momentum scales of $\mathcal{O}(gT)$. Although mechanisms have been suggested which do not require the dynamic generation of a magnetic mass scale, $m_m \sim \mathcal{O}(g^2 T)$, which could cure the remaining infrared divergences² the existence of such a mass would clearly be sufficient³. Assuming the existence of a non-vanishing magnetic mass Rebhan has shown that this will influence the perturbative calculation of the electric mass already at next-to-leading order⁴, *i.e.* at $\mathcal{O}(g^2 \ln g)$.

The analysis of electric and magnetic screening properties in the high temperature phase also is important for our understanding of the nature of fundamental excitations in the QCD plasma phase. Are quarks and gluons the basic degrees of freedom in the plasma phase? Can one give to them a gauge invariant meaning or should one try to understand the plasma phase in terms of colourless excitations only? Calculations of the QCD equation of state clearly suggests that the relevant degrees of freedom in the high temperature phase are those of quarks and gluons. However, to which extent these partonic degrees of freedom do have further dynamic significance in the high temperature phase is not obvious. Can we give, for instance, the thermal electric gluon mass a physical, *i.e.* gauge invariant, meaning? To some extent this has been answered by Kobes et. al.⁵. They show that although the gluon propagator,

$$G_\mu(p_0, \vec{p}) \equiv \langle \text{Tr} A_\mu(p_0, \vec{p}) A_\mu^\dagger(p_0, \vec{p}) \rangle \sim (p^2 + \Pi_{\mu\mu}(p_0, \vec{p}))^{-1} \quad , \quad (1)$$

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is a gauge dependent observable, the pole masses,

$$m_\mu^2 = \Pi_{\mu\mu}(0, |\vec{p}|^2 = -m_\mu^2) \quad (2)$$

are gauge invariant.

In order to circumvent the definition of m_e and m_m in terms of gauge dependent operators attempts have been undertaken to introduce gauge invariant observables for the calculation of gluon screening masses⁶. Here, however, one has to examine in how far the masses extracted from gauge invariant operators correspond to those of *elementary excitations* or to *quasi-particle states* which may result from superpositions of several gluons. This problem became, for instance, apparent in recent studies of the thermal W-boson mass in the symmetric high temperature phase of the electroweak theory⁷.

Eventually we clearly have to aim at an analysis of various operators that allow to extract the thermal gluon masses. In the following we will concentrate on the calculation of electric, $m_e \equiv m_0$, and magnetic, $m_m \equiv m_i$, $i \neq 0$, masses from the gluon propagator in Landau gauge. These pole masses can be obtained from the exponential decay of finite temperature gluon correlation functions at large spatial separations.

2 Electric and Magnetic Screening Masses

We have analyzed the gluon propagator in coordinate space, *i.e.* we calculate spatial correlation functions of static ($p_0 \equiv 0$) gauge fields, $\tilde{A}_\mu(x_3) \equiv \sum_{x_0, x_1, x_3} A_\mu(x_0, x_1, x_2, x_3)$, in the x_3 -direction of finite temperature lattices of size $N_\tau \times N_\sigma^2 \times N_3$. The long distance behaviour of the correlation function, $\tilde{G}_\mu(x_3) = \langle \text{Tr} A_\mu(x_3) \tilde{A}_\mu^\dagger(x_3) \rangle$, yields the electric and magnetic masses, respectively^b. In order to check the influence of discretization errors resulting from the finite lattice spacing, a , we have performed calculations with the standard Wilson action as well as with an $\mathcal{O}(a^2)$ Symanzik-improved action. In addition we have chosen two different temporal lattice sizes, $N_\tau = 4$ and 8 . At fixed temperature $T \equiv 1/N_\tau a$ we thus can perform calculations at values of the lattice cut-off that differ by a factor two.

Let us start with a discussion of the electric screening mass. In Fig. 1 we show m_e/T for both types of actions and the two different temporal lattice sizes. Within errors, m_e/T does not differ significantly for the three cases. Even the Symanzik-improved action, does not shift the electric screening mass

^bThese calculations are an extension of our earlier investigations in a much smaller temperature interval⁸. For details on the definition of $A_\mu(x)$ on the lattice, the gauge fixing and further lattice specific details we refer to Refs. 8 and 9.

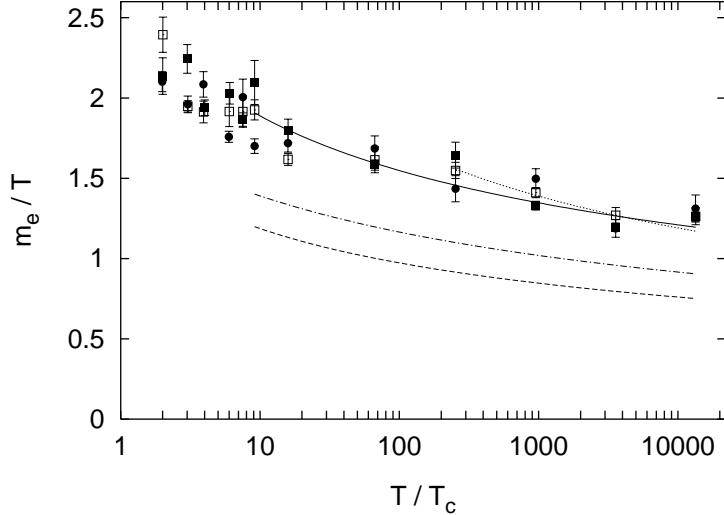


Figure 1: The electric screening mass in units of the temperature versus T/T_c calculated on lattices of size 4×32^3 (filled squares) and $8 \times 32^2 \times 64$ (open squares) with the Wilson action and on a 4×32^3 lattice with the Symanzik improved action (filled circles). The curves show leading order (dashed) and next-to-leading order (dashed-dotted) perturbative results as well as fits with the ansätze given in Eq. (4) (see text for details).

in any direction. This is quite different from what has been observed in calculations of bulk thermodynamic observables like the energy density or pressure¹⁰. It is, however, in accordance with the expectation that the screening masses are entirely dominated by infra-red effects, while the bulk thermodynamic observables receive large contributions from ultra-violet modes, which in turn are strongly influenced by finite cut-off effects.

We have analyzed the temperature dependence of m_e/T using ansätze motivated by the leading and next-to-leading order perturbative calculations

$$\left(\frac{m_e}{T}\right)^2 = \begin{cases} Ag^2(T) & , \text{ case-A} \\ \frac{2}{3}g^2(T) \left[1 + \frac{3}{2\pi} \frac{m_e}{T} \left(\ln\left(\frac{2m_e}{m_m}\right) - \frac{1}{2} \right) \right] + Bg^4(T) & , \text{ case-B} \end{cases} \quad (3)$$

where we use for the running coupling the two-loop β -function

$$g^{-2}(T) = \frac{11}{12\pi^2} \ln\left(\frac{2\pi T}{\Lambda_{\overline{\text{MS}}}}\right) + \frac{17}{44\pi^2} \ln\left[2 \ln\left(\frac{2\pi T}{\Lambda_{\overline{\text{MS}}}}\right)\right] , \quad (4)$$

and relate $\Lambda_{\overline{\text{MS}}}$ to the critical temperature for the deconfinement transition, $T_c/\Lambda_{\overline{\text{MS}}} \simeq 1.08^{11}$.

To make use of the next-to-leading order ansatz we determine also the magnetic mass, m_m . Results for the ratio m_e/m_m obtained from calculations with the Wilson action on a lattice of size $8 \times 32^2 \times 64$ are shown in Fig. 2. The naive expectation, $m_m/m_e \sim g(T)$, does seem to describe this ratio quite well. A fit with such an ansatz for $T \geq 2T_c$ yields

$$\left(\frac{m_e}{m_m}\right)^2 = (7.4 \pm 0.3) g^{-2}(T) . \quad (5)$$

with $\chi^2/\text{dof} = 1.4$. A fit for m_m itself, using the ansatz $m_m/T \sim g^2(T)$, yields

$$\frac{m_m}{T} = (0.46 \pm 0.01)g^2(T) , \quad (6)$$

which is in good agreement with our earlier calculation in a much narrower temperature interval⁸.

In Fig. 1 we show the leading (case-A with $A=2/3$) and next-to-leading order (case-B with $B=0$) perturbative results as dashed and dash-dotted curves, respectively. Even at temperatures $T \sim 10^4 T_c$ the leading order result deviates from the numerical results by nearly a factor of two. The next-to-leading order result clearly yields an improved description. Including an additional $\mathcal{O}(g^4 T)$ correction as suggested in the case-B fit leads, however, to a too strong variation with temperature to provide a good description in the entire temperature interval. This is shown by the dotted curve in Fig. 1. The fit for temperatures $T > 100 T_c$ yields for the coefficient of the $\mathcal{O}(g^4)$ correction $B = 0.54 \pm 0.03$. The case-A fit, on the other hand, does yield a satisfactory description of the data for $T > 10 T_c$. This is also shown in Fig. 1 as a solid curve. For the fit parameter we find $A = 1.69 \pm 0.02$ with $\chi^2/\text{dof} = 4.1$. This, of course, is consistent with the fits from Eq. 5 and 6. The numerical value, however, exceeds the leading order perturbative result by a factor 2.5.

3 Conclusions

The analysis of electric and magnetic screening masses shows that m_e/T as well as m_m/T are *running with temperature*. The temperature variation is consistent with a logarithmic dependence. In particular, we have evidence that $m_m/m_e \sim g(T)$ as expected from general considerations of the infrared-behaviour of high temperature QCD. Quantitative results, however, do not agree with leading and next-to-leading order perturbation theory even at temperatures as high as $10^4 T_c$. Similar conclusions have been drawn from investigations of the SU(2) finite temperature theory in the context of dimensional reduction using a gauge invariant operator^{12,13}.

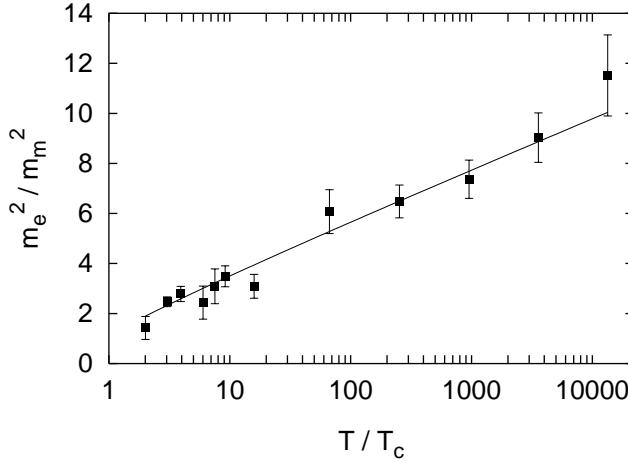


Figure 2: Squared ratio of the electric and magnetic screening masses vs. T/T_c from simulations on a $8 \times 32^2 \times 64$ lattice using the Wilson action.

References

1. see for instance, J. I. Kapusta, Finite Temperature Field Theory, Cambridge University Press 1989.
2. J.-P. Blaizot and E. Iancu, Nucl. Phys. **B459** (1996) 559.
3. A.D. Linde, Phys. Lett. **B96** (1980) 289.
4. A.K. Rebhan, Phys. Rev. **D48** (1993) R3967 and Nucl. Phys. **B430** (1994) 319.
5. R. Kobes, G. Kunstatter, A. Rebhan, Phys. Rev. Lett. **64** (1990) 2992 and Nucl. Phys. **B355** (1991) 1.
6. P. Arnold and L. G. Yaffe, Phys. Rev. **D52** (1995) 7208.
7. F. Karsch, T. Neuhaus, A. Patkós and J. Rank, Nucl. Phys. **B474** (1996) 217.
8. U.M. Heller, F. Karsch and J. Rank, Phys. Lett. **B355** (1995) 511.
9. U.M. Heller, F. Karsch and J. Rank, in preparation.
10. B. Beinlich, F. Karsch and E. Laermann, Nucl. Phys. **B462** (1996) 415.
11. J. Fingberg, U.M. Heller and F. Karsch, Nucl. Phys. **B392** (1993) 493.
12. K. Kajantie, M. Laine, K. Rummukainen and M. Shaposhnikov, hep-ph/9704416, April 1997.
13. K. Rummukainen, contribution to these proceedings.